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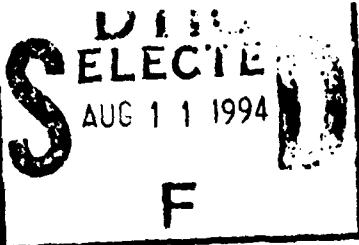
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PREDICTION OF THE PRESSURE LOSS AND DENSITY  
FACTORS FOR TWO-PHASE ANNULAR FLOW WITH  
OR WITHOUT HEAT GENERATION

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# ABSTRACT

A simple analysis is presented for predicting pressure loss and density factors for two-phase, one-component annular flow with and without heat generation. All four combinations of laminar and turbulent flow in both the annulus and core are considered. The analysis is based on assumed velocity profiles and matching velocity and shear stress at the liquid-vapor interface. The theory is found to compare favorably with experimental results on both vertical and horizontal sections. The application of the theory to natural circulation steam generators is also presented.

# PREDICTION OF THE PRESSURE LOSS AND DENSITY FACTORS FOR TWO-PHASE ANNULAR FLOW WITH OR WITHOUT HEAT GENERATION<sup>1</sup>

By J. C. Westmoreland

## NOMENCLATURE

The following nomenclature is used in the paper:

- A - cross sectional area
- a - internal radius
- D - internal diameter
- F - summation of pressure loss factors
- f - moody friction factor over four
- g - gravity constant
- h - enthalpy
- k - specific pressure-loss factor
- n - number of risers
- P - two-phase pressure gradient
- p - fluid pressure
- q - heat transfer rate per unit area
- r - volume fraction of two-phase constituent
- s - internal flow path or stream line coordinate
- V - velocity of fluid
- v - specific volume
- w - weight rate of flow
- x - vapor quality
- $\alpha$  - angle of inclination from vertical
- $\eta$  -  $(1 - \sqrt{rg})$

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$\theta$  - time

$\mu$  - with subscript dynamic viscosity, otherwise viscosity ratio

$\nu$  -  $v_f/v_g$

$\rho$  - fluid density

$\sigma$  - water level swell factor

$\beta$  -  $(1 - v_f / v_g)$

$\tau$  - shear stress

$\phi$  - two-phase flow distribution factor

$\psi$  - recirculation ratio

### Subscripts

d - downcomer liquid

f - saturated liquid

g - saturated vapor

o - equivalent liquid flow for two-phase flow

w - solid boundary condition

Note: Average values of velocity are denoted by a cross bar.

### INTRODUCTION

In the design of two-phase flow systems for a wide field of application, an analytical treatment of the pressure loss and density variations involved would be extremely useful; in some cases where experimental results are lacking or practical experience is missing this treatment becomes quite necessary. Because of the more recent emphasis on the design of nuclear power plants which comprise liquid-cooled reactors and steam generators, a greater importance has been placed on the need for such a treatment in the power generation field. In this regard, the design of the boiling water reactors and nuclear steam generators has presented problems which pertain to their control and performance. The solution of these problems requires a detailed knowledge of the two-phase flow phenomena that occur in the associated steam generation sections. Heretofore, graphical solutions of a completely empirical nature have been employed exclusively in solving these particular problems; however, these solutions have proven impractical for detailed analysis to be made with an extensive variation of operating conditions. These techniques offer little help in attempting to predict the transient behavior of any specific system.

The following analysis of two-phase annular flow has been made to provide a means whereby purely analytical procedures may be employed in predicting the pressure loss and density relations for such flows in systems for which there is a distinct lack of experimental data. Through the use of established mechanics, a general theory for two phase annular flow has been developed for evaluating flow problems of this nature. The analysis, however, represents a proposed method for correlating two-phase data in the appropriate range of volume fractions. The results of the analysis have been employed in successfully predicting the experimental data of a number of investigations; the more general results have been reduced to dimensionless form. The practical application of the theory to a specific case will be presented and the feasibility of the results is being studied with regard to the design of natural circulation steam generators for nuclear power plants.

### TWO-PHASE FLOW ANALYSIS

The basic concept behind the present analysis of two-phase annular flow consists of prescribing a unique criteria in terms of boundary conditions which give the recirculation ratio as an explicit function of only the thermodynamic state and volume fraction in lieu of the particular system geometry parameters and absolute flow rate. This important result is obtained by performing the following integrations:

$$\frac{\delta W_f}{\delta W_g} = \frac{\int_{A_f} \rho_f V_f dA_f}{\int_{A_g} \rho_g V_g dA_g} \quad (1)$$

where  $w_f$  is the liquid annulus mass flow rate and  $w_g$  is the gas core mass flow rate. The boundary conditions for these integrations are zero liquid velocity at the wall and equal velocity and shear stress at the two-phase fluid interface. The velocity distribution for turbulent flow is taken to be the one-seventh law, and for laminar flow, the square law is employed. The basic assumptions for this analysis may be enumerated as follows:

1. We assume constant static pressure across any flow cross section.
2. We assume constant fluid properties across any flow cross section.
3. We neglect the effects of any interphase dispersion or entrainment.
4. We assume saturated generation of vapor.

It has been recognized that each phase may experience a transition from the laminar to the turbulent flow regime and vice versa under different conditions, and the indicated integrations have been made for all permutations of the two possible simultaneous flow conditions. The appropriate velocity distribution for each phase has been selected in accordance with the previously stated criteria. The results\* of these integrations are:

Laminar - Laminar

$$\phi = \frac{(2 - \mu r_2 - (1 - \mu) r_2^2) r_2}{(1 - r_2)^2} \quad (2)$$

---

\*A detailed derivation of the turbulent-laminar case is given in the Appendix.

Turbulent - Turbulent

$$\phi = \frac{\mu^2 \left\{ 7 \left( 1 + \frac{1-\eta}{\mu\eta} \right)^{1/2} - 15 \left( 1 + \frac{1-\eta}{\mu\eta} \right) + 8 \right\}}{\left\{ \frac{15}{\eta} - 8 \right\}} \quad (3)$$

Laminar - Turbulent

$$\phi = \frac{k_g}{(1-k_g)} \frac{7}{30\alpha^2} \left\{ \frac{7}{(1-\alpha)^{1/2}} - 15(1-\alpha) + 8(1-\alpha)^2 \right\} \quad (4)$$

where

$$\alpha = \frac{14k_g}{(14-\mu)k_g + \mu}$$

Turbulent - Laminar

$$\phi = \frac{15}{49} \frac{k_g}{\eta\mu(8\eta-15)} \left\{ \frac{k_g}{\eta(1-\eta)} + 28\mu \right\} \quad (5)$$

where the two-phase flow distribution factor  $\phi$  has been extracted from the result obtained for mass flow ratio

$$\frac{\delta W_L}{\delta W_g} = \frac{1}{\nu\phi} \quad (6)$$

In any one permutation of flows the liquid-phase flow condition is described first. The flow distribution factor is dimensionless and independent of the system geometry parameters, therefore, it becomes only a general function of the thermodynamic state and volume fraction of gas. While the previous derivations were accomplished for a tubular cross section, the results are felt to be representative of, and have been compared favorably with, the flow between two flat surfaces.

The flow regime associated with one phase or the other may be ascertained once we are able to predict the appropriate Reynold's number. This may be done for the gas phase by taking the characteristic velocity, which is descriptive of the ratio of inertia forces to the viscous forces at the two-phase interface, as the excess of average gas velocity over the average liquid velocity. The Reynold's number for the gas phase is then:

$$N_{Re_g} = \frac{(\bar{V}_g - \bar{V}_L) \rho (1-\eta) D}{\mu_g} \quad (7)$$

It becomes convenient to express the ratio of average velocities in dimensionless form; this ratio may be obtained from the results of Equation (6) as follows:

$$\frac{\bar{V}_L}{\bar{V}_g} = \frac{k_g}{(1-k_g)\phi} \quad (8)$$

For the liquid phase we arbitrarily define the Reynold's number in terms of an apparent liquid flow. These terms were derived by integrating the velocity profile in the annulus across the entire cross section. The Reynold's number for the liquid phase then becomes

$$N_{REF} = \frac{\bar{V}_{app} \rho_f D}{\mu_f} \quad (9)$$

In examining the possibility of any one permutation of laminar and turbulent flows occurring in our model, a useful result is obtained by taking the ratio of Equations (9) to (7)

$$\frac{N_{REF}}{N_{REG}} = \left[ \frac{\mu}{\gamma} \right] \left[ \frac{\bar{V}_g}{\bar{V}_f} - 1 \right]^{-1} \left[ \frac{(1-r_g) \int_0^a V_f dA_f}{(1-\eta) \int_0^a V_f dA_f} \right] \quad (10)$$

For any one thermodynamic state it can be shown that at moderate volume fractions of gas ( $r_g = 0.4$ ), the Reynold's number for the liquid phase is much greater than that for the gas phase, and at very high volume fractions of gas ( $r_g = 0.9$ ). The two-phase flow is characterized by the Reynold's number for each phase becoming approximately equal. This observation would indicate that, without prior knowledge of the geometry or flow conditions for any particular system, the most probable configuration of flow regimes would be turbulent-laminar at moderate volume fractions and laminar-turbulent at high volume fractions. This deduction for probable modes of flow has been borne out by the results obtained by Dengler<sup>(1)</sup> on a particular system. In these experiments, photographs of two-phase annular flow were taken which clearly indicate a turbulent annulus and laminar core at volume fraction of gas of approximately 0.20; the observations are distinctly reversed for the photograph shown at higher volume fractions of approximately 0.90.

#### TWO-PHASE PRESSURE LOSS ANALYSIS

The pressure loss which occurs across a flow section with two-phase flow has been derived by superimposing linearly the effects caused by wall friction, gravity, and net momentum flux. In a vapor generation section the assumption of no interphase dispersion is subject to question, and it is obly recognized that consideration should be given to the change in momentum that arises from the formation of vapor bubbles in the liquid film at the wall. Through a vertical section the static pressure is assumed constant across a flow section, and we tacitly employ the same assumption for the inclined or horizontal case. As previously stated, the fluid properties are taken as constant across a cross section.

The local pressure loss caused by wall friction was determined by acknowledging the wall shear stress which resulted from the liquid film in terms of the apparent liquid flow as follows:

$$\delta P_f \sim \tau_w = f_{app} \frac{\rho_f \bar{V}_{app}^2}{2} \quad (11)$$

In terms of the total liquid flow\* we prescribe the frictional pressure loss or wall shear stress in a similar manner

$$\delta P_0 \sim \tau_{w0} \equiv f_0 \frac{\rho_L \bar{V}_0^2}{2} \quad (12)$$

the total liquid flow is considered at the same thermodynamic state as the two-phase flow. By dividing the two previous expressions we obtain

$$\frac{\delta P_f}{\delta P_0} = \frac{f_{app}}{f_0} \left( \frac{\bar{V}_{app}}{\bar{V}_0} \right)^2 \quad (13)$$

where the friction factor  $f_{app}$  is evaluated at a Reynold's number for the apparent liquid flow derived previously. We may proceed to write the general expression for the friction factor in terms of the Reynold's number as

$$f = \frac{C}{(N_{RE})^m} \quad (14)$$

If  $C$  is an empirical constant that represents the friction factor variation for the liquid flows considered when the appropriate flow regime is established, then we may write the local pressure-gradient ratio as

$$\frac{\delta P_f}{\delta P_0} = \left( \frac{N_{REa}}{N_{REapp}} \right)^m \left( \frac{\bar{V}_{app}}{\bar{V}_0} \right)^2 = \left( \frac{\bar{V}_{app}}{\bar{V}_0} \right)^{2-m} \quad (15)$$

By inserting in this expression the appropriate Reynold's number for the apparent liquid flow along with the appropriate number for the total mass flow in the liquid state, we readily derive the following general result for the turbulent annulus

$$\frac{\delta P_f}{\delta P_0} = \left[ \frac{\int_0^a V_f dA_f}{(1 + 2\phi) \int_0^a V_f dA_f} \right]^{(2-m)} \quad (16)$$

The previous expression is dimensionless and independent of system geometry and total flow rates; hence by selecting appropriate values for the exponent  $m$ , we may proceed to obtain dimensionless plots of two-phase friction pressure loss versus the volume fraction of steam at constant thermodynamic states. It has been found that an exponent of  $1/4$  represented extremely well the variation of friction factor with a Reynold's number for the case of flow along a flat plate in the turbulent regime; the same exponent has been found satisfactory for data taken from turbulent tube flow. We have in a laminar flow the well-known exponent of one. It is interesting to note that if we assume the Reynold's number for the annulus flow to be the same as for the total liquid

\*Gas plus liquid.



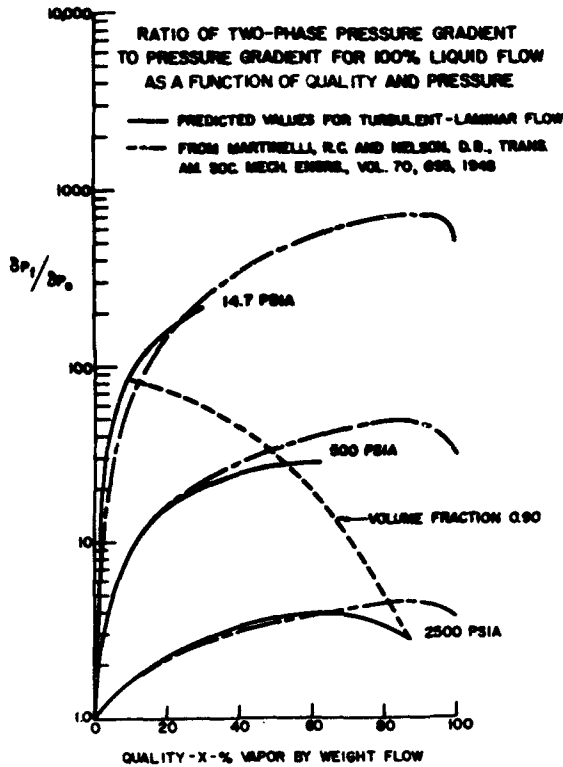


Fig.1

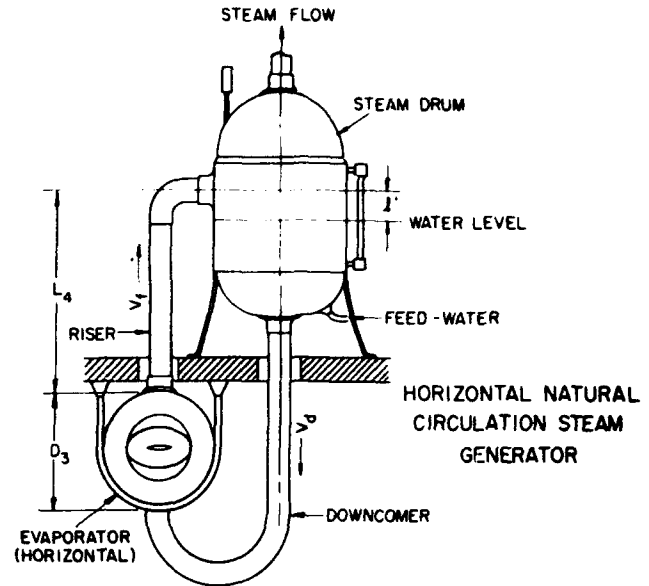


Fig.2

flow, then this exponent becomes 0, and all possible cases may be described with values of  $m$  ranging from 0 to 1. Without knowledge of any one particular system and the associated flow conditions, it can be shown that an exponent of  $1/4$  gives a fairly accurate estimate of the two-phase friction pressure loss. This conclusion was established from the results shown in Figure 1.

For the case in which vapor is being generated along the flow channels, a change in volume fraction of steam is experienced from inlet to outlet. The functional variation of volume fraction with path length may be derived from the heat flux in the following manner:

$$w_g = \int_0^s \frac{q dA}{h_{fg} A} = \frac{w_0}{\psi} \quad (17)$$

The volume fraction,  $r_g$ , at any distance,  $s$ , along the flow path is then determined from the total flow,  $w_0$ , the steam flow above, and the corresponding value of  $r_g$  in the appropriate permutation flow regimes. After the distribution of  $r_g$  in the  $s$  direction has been obtained, the two-phase frictional pressure-drop may be averaged over the path length by the following integration:

$$\frac{P_f}{P_0} = \frac{L \int_0^{x_{ex.}} (\delta P_f / \delta P_0) dr_g}{\int_0^L r_g ds} \quad (18)$$

This result is then employed to predict the frictional pressure loss for heat generation.

The pressure loss attributed to a change in momentum flux may be derived for an incremental section by writing the following expression for this change when the inlet condition is zero volume fraction of vapor and the outlet condition is specified by  $r_g$ :

$$\delta P_{mom.} = \left\{ \frac{\psi^2}{\rho_f} - \frac{1}{\rho_g r_g} - \frac{(\psi-1)^2}{(1-r_g)\rho_f} \right\} \frac{w_g^2}{gA^2 \delta s} \quad (19)$$

This result may be written in terms of a multiplying factor  $R$ , as follows:

$$\delta P_{mom.} = R \left\{ \frac{w_g^2}{gA^2 \rho_f \delta s} \right\} \quad (20)$$

where

$$R = \left\{ \frac{(1-\psi)^2}{(1-r_g)\psi^2} + \frac{1}{\psi^2 r_g} - 1 \right\} \quad (21)$$

The expression  $R$  is dimensionless and with the previous result for the two-phase mass flow ratio

$$\psi = \left( 1 + \frac{1}{\gamma \phi} \right) \quad (22)$$

it becomes an explicit function of volume fraction of steam and thermodynamic state. With the total heat generation specified then the change in momentum flux is determined from the exit volume fraction of vapor.

The pressure gradient required to balance the effect of gravity may be determined for the general case of heat generation and/or variation in cross-sectional area by simply integrating the gravitational component between terminal points

$$\delta P_{grav.} = \frac{\int_0^{\delta s} (1 - \beta r_g) \rho_f \sin \alpha \, ds}{\delta s} \quad (23)$$

where  $\alpha$  is the angle of inclination from the vertical and is a known calculable function of the path-length  $s$ . This result is governed by the recirculation ratio which in turn fixes the volume fraction of gas at any section for the case being studied. For one-component heat generation we obtain the recirculation ratio at any cross section from

$$\psi = \frac{W_0}{\int_0^{\delta s} \frac{Q dA}{h_{fg} A}} \quad (24)$$

from which the volume fraction of vapor may be determined through the results of the previous two-phase flow analysis.

The total pressure loss for two-phase flow is then obtained by summing the previous results to give

$$\delta P_{tot} = \sum_0^i \delta P_i \quad (25)$$

This result is known once the geometry of the system is determined, and when appropriate, the heat generation characteristics are specified along with the total liquid flow. To predict the pressure loss associated with the flow through valves and fittings, and also the losses in the entrance and exit regions of channels, the velocity-head concept is invoked. The appropriate single-phase pressure loss factor being applied to the total mass flow is

$$\delta P = K \frac{\rho \bar{V}^2}{2} = \left\{ \frac{(1 + \nu \phi)^2 (1 - \beta_{fg})^2}{(1 - \beta_{fg})} \right\} \frac{K \rho_f \bar{V}_f^2}{2} \quad (26)$$

where K is the single-phase loss factor,

$$\rho = (1 - \beta_{fg}) \rho_f \quad (27)$$

and

$$\bar{V} = \frac{W_0}{(1 - \beta_{fg}) \rho_f A} \quad (28)$$

In the absence of a better concept to use in predicting such losses, this result seems reasonable. Perhaps as the state of the art of two-phase flow is further advanced, appropriate empirical correlations will become available that describe this pressure loss more accurately.

#### APPLICATIONS

The present analysis has been employed effectively in analyzing the dynamics of natural circulation steam generators for nuclear power plants. A schematic diagram of this unit is shown in Figure 2 and the major components are riser, downcomer, evaporator, and steam drum. The circulation of fluid is in the direction indicated and is established by the generation of steam in the evaporator, which in turn causes the gravity term in the riser to diminish and so creates an unbalance with the downcomer gravity term. This condition results in a circulation of magnitude determined by the frictional and momentum losses throughout the flow path. The water level maintained in the steam drum is one of the independent variables along with the magnitude of the circulation. To solve this problem we sum the pressure losses around the flow circuit and equate the result to zero

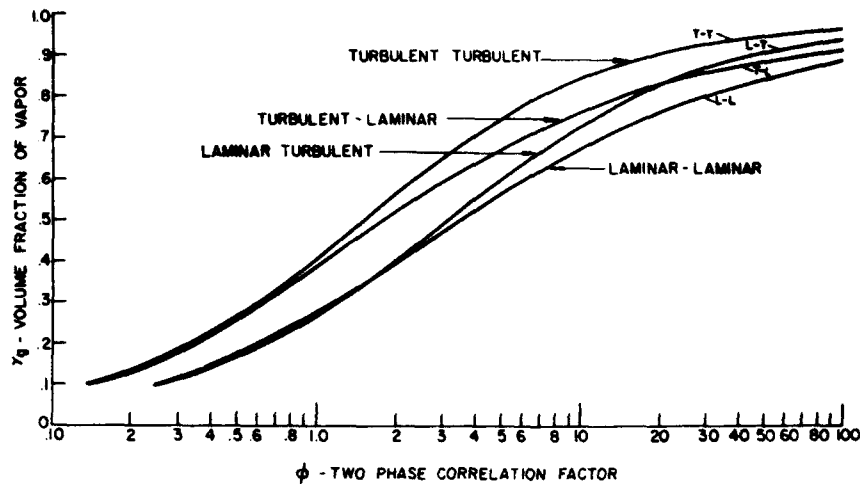
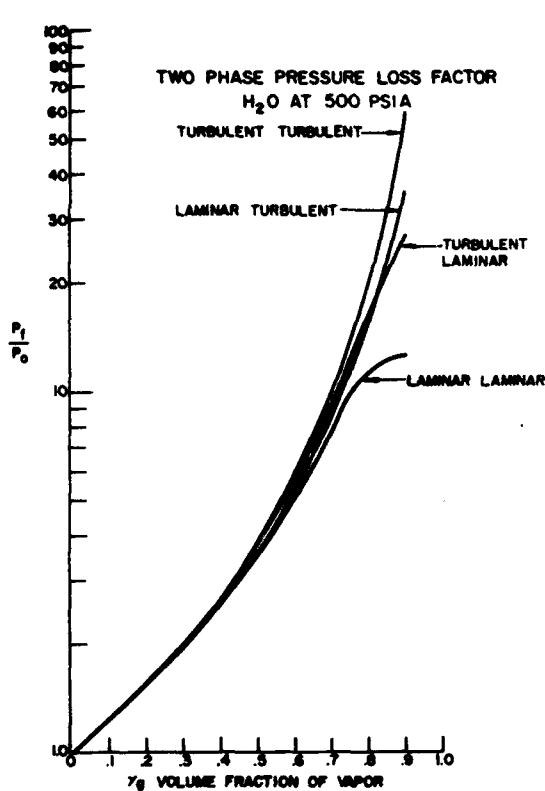
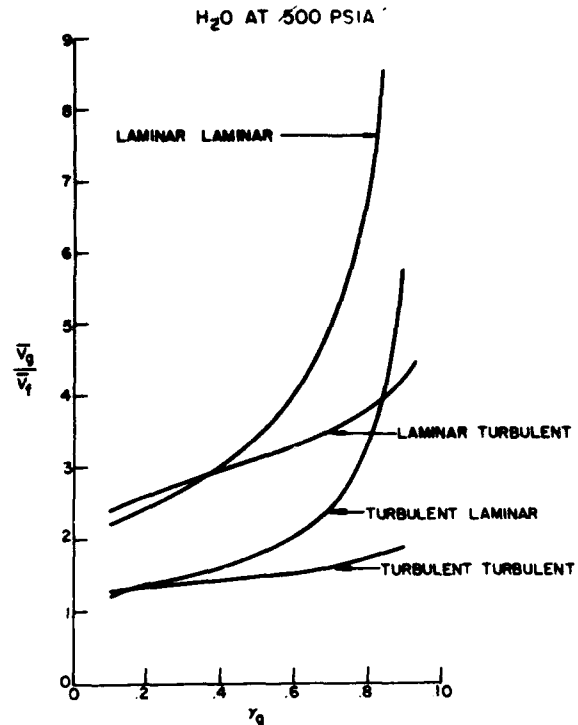
Fig. 3 Phase flow distribution factor H<sub>2</sub>O at 500 psia

Fig. 4

Fig. 5 Two-phase slip velocity ratio H<sub>2</sub>O at 500 psia

$$\sum \Delta P = 0$$

(29)

In this investigation it becomes necessary to obtain knowledge of the transient behavior of these units. It was found that the solution for the transient case could be employed effectively for steady-state solutions simply by placing any reasonable value of the independent variable (water level) into the transient

solution; the result would be the correct value for the steam flow and thermodynamic state. The latter values are obtained from the solution to the energy equations for the system.

The application of conservation of momentum around the steam generator

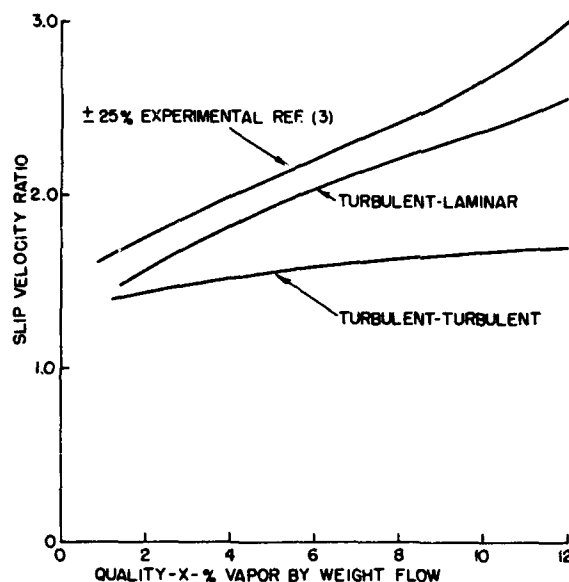


Fig.6 Slip velocity ratio prediction  
H<sub>2</sub>O at 615 psia  
Exit values of a vertical heat generation  
section

circuit and the zero summation of pressures around the closed loop is next considered. The result becomes, for the case of a turbulent riser flow,

$$\left\{ F_1 + \frac{4fL}{D} \right\} \frac{\rho_1 \bar{V}_1^2}{2g} + (1-x_1) \frac{\epsilon_1 L_1 \rho_1}{g} \frac{d\bar{V}_1}{d\theta} + F_2 \frac{\rho_2 \bar{V}_2^2}{2g} + \frac{\epsilon_2 L_2 \rho_2}{g} \frac{d\bar{V}_2}{d\theta} \quad (30)$$

$$= \left[ \left( L_1 + \frac{D_1}{2} \right) \beta_1 x_1 + \beta_2 L \right] \rho_1$$

$$\frac{4fL}{D} = \left\{ \left( \frac{\rho_1}{\rho_0} \right) (1+x\phi)^2 (1-x_1)^2 4f_0 \frac{L}{D} \right\} \quad (31)$$

and  $F_1$  is the summation of two-phase k factors for the riser flow circuit.

In taking a mass balance on the water in the steam drum with the condition of matched feedwater flow, we obtain

$$W_d = W_0 + W_f + \rho_f A_1 \frac{dL}{d\theta} \quad (32)$$

The two-phase flow in the riser is solved by

$$W_f = \frac{W_g}{\nu \phi} \quad (33)$$

The solid water steam drum water level parameter,  $l$ , is obtained from the volume fraction of steam in the risers by

$$l = L_0 - \delta h \quad (34)$$

Equations (30), (31), (32), and (33) have been combined to give the volume fraction of steam,  $r_g$ , as a function of the steam pressure and flow rate, and also derivatives thereof. The latter variables are employed as known parameters inasmuch as the solutions to the energy equations for the system are not the subject under consideration. With these terms as driving functions, the finite difference method has been used to obtain a numerical solution of the resultant differential equation by means of the IBM-650 digital computer.

The results of the previous analysis have been employed to study practical steam generators as the design indicated; the comparison between theory and data has been very good.

### RESULTS

The two-phase flow distribution factor,  $\phi$ , computed for the annular flow of saturated water and steam at 500 psia, is shown in Figure 3 as a function of the volume fraction of steam  $r_g$ . The four possible permutations of the laminar and the turbulent flow regimes are indicated, and it is interesting to note that for the laminar-turbulent case and also the turbulent-laminar case the water annulus dominates the magnitude of  $\phi$  at low volume fractions with the steam core dominating at high volume fractions. This latter observation is made when the same flow regime persists in both the annulus and the core. As the thermodynamic state influences  $\phi$  through the ratio of viscosities for steam and water, it can be shown that this effect may be neglected for some applications over a wide range of pressure.

The two-phase local pressure loss factor,  $\delta P / \delta P_0$ , computed for the annular flow of saturated water and steam at 500 psia, is shown in Figure 4 as a function of the volume fraction of steam,  $r_g$ . Again, the four possible permutations of flow regimes are indicated. These results show that for this situation the four curves are quite similar up to a volume fraction of approximately 0.70, and thereafter the laminar-laminar case and the turbulent-turbulent case become divergent in opposite directions. In this case of two-phase friction loss,  $\delta P / \delta P_0$ , the thermodynamic state has a strong influence on the magnitude of the results.

The two-phase slip velocity ratio computed for the previous conditions is shown in Figure 5 as a function of volume fraction of steam. The discussion given for the results of the two-phase flow distribution factor  $\phi$ , shown in Figure 3, applies identically to this factor with regard to its behavior characteristics. Also shown in Figure 6 is the empirical curve proposed by Cook<sup>(2)</sup> which was obtained from experimental data taken in a vertical rectangular section (1/2 in. wide, 2 in. deep and 2 ft. long - approximate dimensions) for water flowing at 600 psi with uniform heat generation. The

volume fraction of steam was determined by the radioactive tracer method and the correlation is reported to be good within  $\pm 25\%$ . In the discussion given previously with regard to the relative independence of slip velocity with density, the comparison of the analytical results with the experimental results at 600 psia (the viscosity discrepancy has negligible effects in this case) is considered excellent for the estimated turbulent-laminar and turbulent-turbulent flow regime permutations.

For the two-component flow of a liquid and a gas, the analytical results have been employed in predicting the experimental data.<sup>(3)</sup> In these experiments the concurrent flow of a liquid-gas mixture in a pipe-line contactor was studied and both oil-air and water-air combinations were considered. The measurements taken were pressure-loss, mass flow rates, and static volume fraction of gas. Photographs of the two phase flow were taken in glass flow sections in order that the different types of flow could be observed.

In employing the analysis to predict these data we could either start with each of the two mass flow rates or the volume fraction being known and then proceed to compute the remaining variables. A comparison of the analytical results with three sets of experimental data is presented in the table with the volume fraction of liquid being taken as the known variable for the analysis. The comparison is considered good although the change in density of the air as it passed through the contactor was neglected since only the average volume fractions were reported for the experiments. To distinguish between the four possible permutations of flow regimes the Reynold's numbers based on the superficial velocities were used as a rough guide. It was reported that annular flow occurred for the experiments at volume fractions of gas from 0.50 to 0.95 where the flow path occurred principally in the horizontal plane. The analytical results are tentatively restricted to volume fractions of gas from 0.20 to 0.90 for vertical flow, and the question arising from the assumption of constant static pressure across a flow section would leave some doubt in regard to this range being applied to the horizontal case.

In general, the analytical results for the two-phase pressure loss factor,  $\frac{\Delta P}{\Delta P_0}$ , have been employed in computing this term versus the vapor quality based on the weight rate of flow at pressures of 14.7, 500, and 2500 psia. These results are shown in the curves of Figure 1 superimposed on the same curve proposed by Martinelli and Nelson.<sup>(4)</sup> It is to be noted that the analytical results are based on the turbulent-laminar permutation of flow regimes. A dashed line is drawn across these curves to indicate when a volume fraction of vapor of 0.9 occurs for the results calculated from Equation (B-20). It is tentatively proposed that the annular flow model begins to deteriorate in this range. The agreement between these two sets of curves is considered more than fortuitous in that the empirical curves were based on the data of Martinelli and Nelson;<sup>(4)</sup> the condition of annular flow is only implied and not specifically stated. It is not surprising to note that at pressures around one atmosphere, the analytical results are applicable only at very low vapor quality based on the weight rate of flow although the static volume fraction of vapor is high at about 0.9.

Prediction of Two-Component Two-Phase Flow Air-Water Mixture

$(r_g)_{avg}$	Experimental Values <sup>(3)</sup>				Predicted Values			
	X	$\psi$	$P_p^*$		X	$\psi$	$P_p^*$	
			$P_o$	$V_G/V_L$			$P_o$	$V_G/V_L$
0.794	1.85	0.0291	13.1	3.30	2.39	0.0192	22.8	2.28
	1.72		12.9		2.39		22.8	
	1.55		16.5		2.39		22.8	
	1.36		16.8		2.39		22.8	
0.501	12.6	0.00305	2.65	1.51	10.14	0.00364	3.99	1.85
	11.9		2.75		10.14		3.99	
	11.0		3.07		10.14		3.99	
	9.92		3.76		10.14		3.99	
0.896	0.365	0.130	67.6	10.3	0.593	0.0713	81.2	5.75
	0.351		72.9		0.593		81.2	
	0.335		81.0		0.593		81.2	
	0.316		84.2		0.593		81.2	

\*In the apparatus there were four passes, and pressure drop measurements were made in each one. Where  $\Delta P_p$  is the two-phase pressure loss and the subscripts G and L refer to the gas and liquid phase, respectively, and X is the Martinelli parameter.

CONCLUSIONS

The success of the annular flow theory in predicting two-phase flow phenomena accurately where there is a liquid film along the wall, but the mid-stream conditions in no way resemble the orderly distribution proposed in the previous analysis, may be attributed to the following postulation. For every case of two-phase flow where the confining surface is wetted by the liquid phase, corresponding hypothetical two-phase annular flow exists which is characterized by a uniform annulus of liquid and a core of gas or vapor. Further, the appropriate velocity distribution may be assigned to both the annulus and the core with the boundary conditions at the two-phase interface being prescribed by equal velocity and shear stress. This postulate is useful when we apply this analysis to the case of two-phase flow in the entrance region of a non-heat generation section where we would not normally expect a fully developed velocity profile to exist until some distance downstream. Although such profiles are utilized in the analysis, we take refuge behind the previous postulation in that the derived two-phase distribution factor represents, in essence, a statement of the condition of flow continuity. Although the flow section may not be long enough to afford fully developed flow, the proper volume fraction of gas and slip velocity ratio will be given. The analytical results for the



two-phase friction pressure loss are subject to modification in this particular case. We would also have to apply similar reasoning in the case of horizontal two-phase flow where because of the effect of the heavier liquid annulus the film thickness is greater at the bottom of the section than at the top. In summary, the two-phase posulate represents a unique criterion for the practical liquid film two-phase flow in that it may be characterized by an idealized annular flow of the type described.

In Figure 7 the two-phase flow configurations of mixed flow and annular flow are shown. In mixed flow two distinct regimes may be characterized as

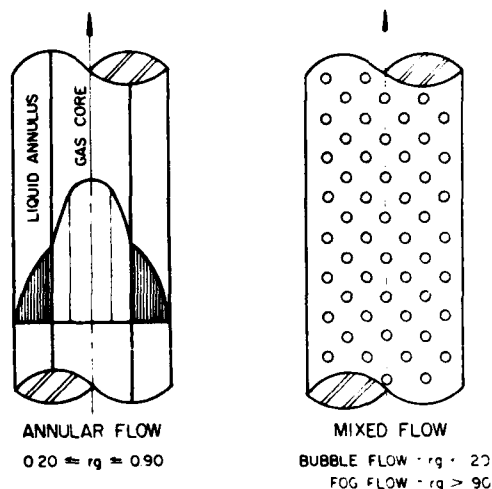


Fig.7 Two-phase flow configurations

follows: for volume fractions of gas less than 0.20, bubble flow occurs with bubbles of gas uniformly dispersed in a parent liquid flow; for volume fractions of gas greater than 0.90 fog flow occurs with spheroids of liquid uniformly dispersed in a parent gas flow. Annular flow is assumed to occur between volume fractions of gas between 0.20 and 0.90, and although it is quite possible that in some cases a permutation of bubble flow and fog flow will occur with an annulus of bubble flow and a core of fog flow the effects of interphase dispersion have been neglected in the analysis. As the volume fraction of gas is an important variable in the study of two-phase flow, it is necessary to distinguish between the static volume fraction, as defined here, and a similar term frequently used which is derived from the ratio of the volume rates of flow.\*

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\*The difference being attributed to the effect of slip-velocity which can be of some magnitude.

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#### APPENDIX. DERIVATION OF TWO-PHASE TURBULENT LAMINAR FLOW

In this specific case the liquid annulus is taken to be in the turbulent regime; the gas core is laminar. The velocity profile for the liquid phase is taken in terms of the  $y$  coordinate being the familiar one-seventh variation.

$$V_f = V_{fmax} \left( \frac{y}{a} \right)^{1/7} \quad (B-1)$$

The velocity profile for the gas phase is taken in terms of the  $r$  coordinate in accordance with the square law

$$V_g = V_{gmax} \left[ 1 - \left( \frac{r}{a_g} \right)^2 \right] \quad (B-2)$$

The total liquid mass flow is determined from

$$\delta W_f = 2\pi a^2 \rho_f V_{fmax} \int_{(1-\frac{a_g}{a})}^0 \left( 1 - \frac{r}{a} \right) \left( \frac{y}{a} \right)^{1/7} d\left( \frac{y}{a} \right) \quad (B-3) \quad \checkmark$$

The total gas flow is established from a similar integration

$$\delta W_g = 2\pi a_g^2 \rho_g V_{gmax} \int_0^{a_g/2} \left[ 1 - \left( \frac{r}{a_g} \right)^2 \right] \left( \frac{r}{a_g} \right) d\left( \frac{r}{a_g} \right) \quad (B-4)$$

The boundary conditions necessary to eliminate the maximum velocity terms are established by taking  $a_g$  in terms of the volume fraction of gas

$$a_g = (1-\eta)a$$

at the wall  $y = 0$

$$V_f = 0 \quad (B-5)$$

at the two-phase interface  $y = a - a_g$

$$V_f = V_g \quad (B-6)$$

and

$$\mu \frac{\partial V_f}{\partial y} = \mu_g \frac{\partial V_g}{\partial r} \quad (B-7)$$

The coordinate,  $a_0$ , where the gas velocity goes to zero is also eliminated by the previous boundary conditions. By performing the indicated integrations and by dividing the mass flow rates, we obtain

$$\frac{\delta W_f}{\delta W_g} = \frac{1}{\nu \phi} \quad (B-8)$$

where

$$\phi = \frac{15}{49} \frac{r_0}{\eta \mu (8\eta - 15)} \left[ \frac{r_0}{\eta (1-\eta)} + 28\mu \right] \quad (B-9)$$

The total flow rate is then simply the sum of the gas and liquid flow

$$W_0 = (1 + \nu \phi) \delta W_f \quad (B-10)$$

Now, for the local pressure loss caused by wall friction, we have the wall shear stress given by

$$\tau_w = f_{app} \frac{\rho_f V_{app}^2}{2} \quad (B-11)$$

where  $V_{app}$  is the average velocity for the apparent liquid flow derived from integrating the velocity profile in the annulus across the entire cross section. The friction factor ( $f_{app}$ ) is based on the Reynold's number of the apparent liquid flow as follows:

$$N_{Re_{app}} = \frac{V_{app} \rho_f D}{\mu} \quad (B-12)$$

At the same section we define a local pressure loss  $\delta p$  which represents the

APR 3 1961

gradient obtained at the same thermodynamic state with an equivalent total liquid flow equal in magnitude to that of the two-phase flow. In the turbulent regime the friction factor is given by

$$f = \frac{0.133}{(N_{Re})^{0.25}} \quad (B-13)$$

The local pressure loss at any section is then given by

$$\delta P_f = 4f \left( \frac{\delta s}{D} \right) \frac{\rho_f \bar{V}_{app}^2}{2} \quad (\text{two-phase pressure loss}) \quad (B-14)$$

$$\delta P_0 = 4f_0 \left( \frac{\delta s}{D} \right) \frac{\rho_f \bar{V}_0^2}{2} \quad (\text{equivalent total liquid flow}) \quad (B-15)$$

By dividing these two expressions and by substituting for the appropriate friction factor we obtain

$$\frac{\delta P_f}{\delta P_0} = \left( \frac{N_{Re_0}}{N_{Re_{app}}} \right)^{0.25} \left( \frac{\bar{V}_{app}}{\bar{V}_0} \right)^2 \quad (B-16)$$

By substituting for the appropriate Reynold's number we obtain

$$\frac{\delta P_f}{\delta P_0} = \left( \frac{\bar{V}_{app}}{\bar{V}_0} \right)^{1.75} \quad (B-17)$$

The apparent liquid velocity  $\bar{V}_{app}$  is obtained from the following integrations:

$$\bar{V}_{app} = 2\pi V_{fmax} \int_0^1 \left(1 - \frac{y}{a}\right) \left(\frac{y}{a}\right)^{1/7} d\left(\frac{y}{a}\right) \quad (\text{equivalent total liquid flow}), \quad (B-18)$$

$$\bar{V}_0 = 2\pi V_{fmax} \left[ \int_0^\eta \left(1 - \frac{y}{a}\right) \left(\frac{y}{a}\right)^{1/7} d\left(\frac{y}{a}\right) \right] (1 + \nu\phi) \quad (B-19)$$

By performing these integrations and by making the appropriate substitutions we obtain

$$\frac{\delta P_f}{\delta P_0} = \frac{1}{(1 + \nu\phi)^{1.75}} \left\{ \frac{7}{15\eta^{8/7} - 8\eta^{15/7}} \right\}^{1.75} \quad (B-20)$$

The local two-phase pressure loss is then given by this expression once the reference pressure loss  $\delta P_0$  is determined.